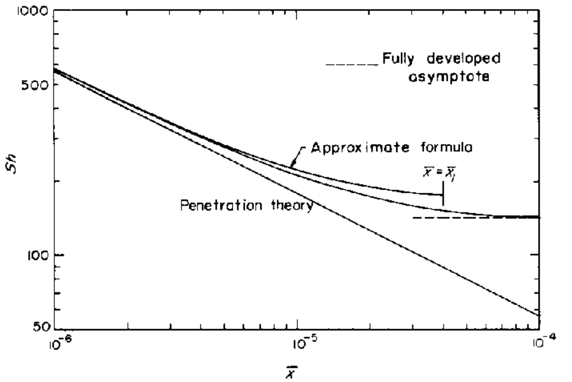

FIG. 2. Comparison of approximation formula, equation (14) with exact result for $\beta = 50\,000$.

Equation (12) predicts the Sherwood number to within a maximum deviation of 2·5 per cent for $\bar{x}$ varying from zero up to the entrance length over the entire range of β investigated ($500 < \beta < 500\,000$). On the scale of Fig. 2, equation (12) can not be distinguished from the numerical solution.

The mass transfer coefficient given by equation (12) may be integrated over the film length to give an expression for the mean Sherwood number based on the average mass transfer coefficient

$$Sh_m = 1{\cdot}128/\sqrt{\bar{x}} + 0{\cdot}1317\beta\sqrt{\bar{x}}. \quad (13)$$

Since equation (12) is valid up to the entrance length, it may be used to predict the entrance length. Substituting for the Sherwood number in equation (12) as 1·05 times the fully developed Sherwood number as given by equation (6) and solving for the entrance length gives an equation for the entrance length.

$$\bar{L}_e = 2{\cdot}56/\beta. \quad (14)$$

As was mentioned previously, the eddy diffusivity given by equation (1) is strictly valid only in the surface region; however, it was used over the entire film thickness. The magnitude of the error incurred as a result of this assumption was estimated by carrying out the integration of equation (10) using a more realistic distribution in the bulk liquid. The eddy diffusivity expression of Reichardt [9] was used for this calculation. For the absorption at 25°C of carbon dioxide in a vertical falling film of water with a Reynolds number of 5000 ($\beta = 54\,700$) it was found that the maximum difference in the calculated Sherwood numbers was 0·7 per cent. This maximum deviation occurred for the fully developed Sherwood number. Thus it is seen that, as expected, the use of equation (1) in the bulk liquid does not result in any significant error.

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INERT GAS EFFECTS IN THE INCIPIENT BOILING OF ALKALI LIQUID METALS

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POOL-BOILING ARTIFICIAL-CAVITY EXPERIMENTS

INCIPIENT boiling superheats of alkali metals measured by different experimenters, and usually even by the same experimenter, are sensitive to small variations in experimental conditions, such as the concentrations of trace contaminants, the presence or absence of small amounts of entrained or dissolved gas, and the prior history of the system up to the point where the first bubble is produced. Even with considerable precautions, the boiling of alkali metals from natural surfaces at low heat fluxes tends to be quite unstable, with large resulting variations in the incipient

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boiling superheat. However, Schultheiss [1] has demonstrated that one can obtain extremely stable boiling with reproducible initial superheats by using a single artificial cavity in a carefully-controlled pool boiling system. Temperature gradients in the liquid were eliminated by maintaining the entrie pool in an isothermal furnace. The system was essentially pressured only by its own vapor pressure, small amounts of argon cover gas in the system being detectable by the change in the condenser heat transfer coefficient. The amount of dissolved oxide, which was controlled by means of the cold-trap temperature, was one of the variables studied. Boiling was reproducibly initiated from the drilled cylindrical cavity at the superheat given by the Laplace equation, where the radius of the cavity is used to determine the superheat vapor pressure difference. Similar results have long been known for pool boiling of nonmetallic liquids from artificial non-wetted cavities, and it has been previously demonstrated that the presence of drilled cavities stabilizes the boiling of liquid metals [2]. What is remarkable about the Schultheiss work is that, based upon measurements of the wetting properties of sodium at elevated temperatures for various metals [3], one might expect the artificial cavity to be fully wetted prior to the initiation of boiling. This is in contradiction to previous theory [4], according to which large superheats are necessary to initiate bubble growth from a well-wetted primary cavity.

We are thus led to consider the possibility in connection with this series of experiments that a small amount of inert gas was stably trapped within the cavity for a long period of time. The equilibrium equation for a meniscus in a cylindrical cavity containing a small amount of inert gas is given by

$$p_L = p_V + p_G - \frac{2\sigma\cos\theta}{r}. \tag{1}$$

For the conditions of these experiments, which corresponded to a liquid head of about 10 cm, the required inert gas partial pressure for a stable meniscus was about 0·015 atm, even assuming perfect wetting. Assuming that the sodium became saturated with argon cover gas during the loading process, this would mean that a small fraction remaining as residual dissolved gas would be sufficient to stabilize the presence of a gas phase in the cavity. It is well known that it is difficult to remove all traces of dissolved gas by boiling in a closed system, even after repeated purging. One notes that this behavior would not be exhibited by natural cavities, which are several order-of-magnitudes smaller than these artificial cavities, since the required gas partial pressure, in order to achieve a stable trapped vapor gas phase, is correspondingly several magnitudes greater. The implications of this hypothesis are that artificial-cavity incipient-boiling superheat measurements can give very little information concerning the results to be expected from natural surfaces.

Another piece of evidence in favor of the above hypothesis is that some of the superheats measured by Schultheiss were actually smaller than would be predicted by equation (1) in the absence of inert gas. Discounting the possibility of contact angles in excess of 100° on the flat metal surfaces, there seems to be no other explanation than the presence of small amounts of dissolved inert gas in the liquid.

OTHER INERT GAS EFFECTS

It is of interest to develop the theory of nucleation by entrained gas bubbles further, in view of the fact that these gas bubbles are stable over a range of vapor pressures, in contrast to pure vapor bubbles. Thus, the equilibrium condition for a gas-containing bubble, given by equation (1) with $\theta = 0°$, together with the Clausius–Clapeyron equation, can be used as the starting point to obtain the equilibrium relationship for a bubble containing a constant number of moles of inert gas:

$$\Delta T = \frac{\gamma}{r} - \frac{\beta T_b}{r^3} \tag{2}$$

where $\Delta T = T_b - T_s$ is the bubble superheat; r is the bubble radius; $\gamma = 2\sigma T_s/\lambda\rho_v$; $\beta = 3n_g R T_s/4\pi\lambda\rho_v$; n_g is the (constant) moles of inert gas; and $\lambda\rho_v$ is the latent heat content per unit volume of vapor at the saturation temperature.

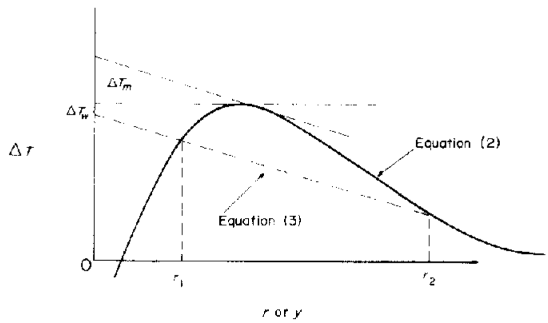

FIG. 1. Stability plot for gas bubble at a heated wall.

This equation, shown as the solid line in Fig. 1, indicates that there is a maximum bubble superheat, $\Delta T_m = T_{bm} - T_s$, at which a bubble containing a given amount of gas can exist in equilibrium with the surrounding liquid. However, below this temperature the bubble may co-exist stably with the liquid at more than one bubble radius for each liquid temperature. Thus, the bubble is stable to small temperature perturbations below T_{bm}, with respect to both growth and collapse, but is unstable to positive temperature fluctuations at $T_b = T_{bm}$. The dotted line represents the temperature profile in the liquid next to the wall, assuming pure conduction heat transfer:

$$\Delta T = \Delta T_w - \frac{qy}{k} \tag{3}$$

where ΔT_w is the wall superheat, q is the wall heat flux, k is the liquid thermal conductivity, and y is the distance from the wall. The intersection of this line with the equilibrium radius line gives the stable bubble radii (assuming no inert gas mass transfer) which a bubble diffusing from the circulating liquid with a given initial volume (and hence a given gas content) will assume in the thermal boundary layer next to the wall. Note that two bubble radii are theoretically possible over a range of wall superheats and heat fluxes. As the wall superheat increases at constant heat flux, a tangency condition gives the threshold condition beyond which bubble equilibrium is no longer possible, resulting in nucleation of a growing vapor-gas bubble. It is readily shown that this tangency condition is given by

$$-\frac{q}{k} = \left(\frac{3\beta T_b}{r^4} - \frac{\gamma}{r^2}\right)\Bigg/\left(1 + \frac{\beta}{r^3}\right). \tag{4}$$

For large k, this reduces to $\Delta T_w = \Delta T_m$, implying that nucleation occurs whenever the wall temperature reaches T_{bm}. If one assumes a plentiful supply of entrained gas bubbles all of the same radius, incipient-boiling will occur reproducibly close to the point where $\Delta T_w = \Delta T_m$. From equation (4) one obtains

$$r_m = (3\beta T_{bm}/\gamma)^{1/2} \quad (5)$$

whence equation (2) gives

$$\Delta T_m \cong \frac{8}{9\lambda\rho_g}\left(\frac{2\pi\sigma^3 T_s}{n_g R}\right)^{1/2} \quad (6)$$

indicating that the wall superheat at which bubble nucleation can occur is inversely proportional to the square root of the initial mass of gas in the bubble. Statistical scatter between observed threshold superheats for inception of boiling in successive experiments may thus be related to variations in bubble radius reaching the wall near the tube exit.

3. CONCLUDING REMARKS

It is our general belief that inert gas effects play a very powerful role in determining incipient boiling superheat in alkali metal loops. Although the physical laws involved are relatively simple, the interactive effects may be complex and may mask other variables. Two examples can be cited: (1) It has been reported [5] that the incipient boiling superheat for sodium in turbulent channel flow exhibits a dependence on the rate of temperature rise, with the inlet temperature increasing in the order of 1°F per minute. A brief consideration of the thermal time constants of the system indicates that a quasi-steady state prevails at all times with respect to temperatures of the system, but the transfer of gas into or out of the surface cavities is diffusion-controlled, and exhibits much longer time constants. Hence, one would suspect some inert-gas effects here. (2) It is important to note that the cavities which remain unflooded over a long period of time in the presence of suspended gas bubbles in the circulating stream depend upon several variables, such as the diameter of the gas bubbles, the cavity diameter, and the contact angle within the cavity. It has been suggested that cracks of the order of 10^{-2} cm width would be stable nucleation sites in a power reactor during a loss-of-flow accident. It is well known, however, that in a mixture of small and large gas bubbles in a liquid, the small bubbles tend to dissolve and the large bubbles tend to grow, owing to surface tension effects. The tendency for gas to enter or to leave a surface cavity will depend upon the distribution of bubble radii in the entrained gas as well as the meniscus curvature, which in turn depends upon the cavity shape, cavity radius, and contact angle. If the meniscus radius of curvature lies in the range of the entrained gas bubble distribution, the smaller bubbles will tend to transport gas to the cavity while the larger bubbles will receive gas from it. The net effect will depend upon a complex rate process. The usual surface cavities, which have a characteristic dimension of 10^{-4} cm, would almost certainly be flooded, in view of the coalescence of suspended bubbles, and the difficulty of producing small, high-pressure bubbles by entrainment or precipitation. Reentrant or sharply-necked cavities will be quite stable in the presence of entrained gas bubbles, since the meniscus is flat in one position at the narrow throat. The end result of these considerations, however, is that natural cone-shaped or cylindrical cavities or cracks between adjoining surfaces may fill with liquid after long periods of operation, even if entrained gas bubbles are present. These bubbles would, however, themselves serve as nucleation centers in the event of a power excursion. The desirability of having a plentiful supply of entrained gas in the circulating stream is therefore indicated.

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